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Analysis of Fiber Bragg Gratings Apodized with Linearly Approximated Segmented Gaussian Function

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Abstract – Typical approaches in writing apodized fiber Bragg gratings (FBG) use approximated profiles such as Gaussian or Raised Cosine distributions. The typical approximation approaches are complicated to be implemented and affect the characteristics of the fiber Bragg gratings especially on the side lobes. In this paper, we report the study on the piecewise stepped approximated Gaussian apodization profile, a new approach proposed to relax the requirement for FBG fabrication setups. The results show no detrimental effects on the side lobes.

Keywords-Apodization, fiber Bragg grating, side lobe suppression ratio.

I. INTRODUCTION

The main peak of uniform Fiber Bragg Gratings in the reflectivity spectrum is accompanied with a series of side lobes in both sides. For some applications like the dense wavelength division multiplexing (DWDM) systems, it is important to lower the reflectivity of the side lobes. A process widely used to achieve this is apodization [1,2]. Apodization is accomplished by varying the amplitude of the coupling coefficient of the forward and backward propagating modes along the length of the grating. The envelope of the amplitude of the coupling coefficient which is called the apodization profile typically follows familiar functions such Gaussian, Raised cosine, and Tanh. However, since it is practically very difficult to follow the ideal functions exactly, certain approximation has to be performed. This

approximation has its own adverse effects on the performance of the Bragg grating, particularly on the SLSR [6,7].

In the following we present a step-wise-linear apodization profile based on Gaussian function that maintains the same suppression of the side lobes and relax the implemented in most of the fabrication setups.

II. THEORY

Many methods are developed for field analysis of the wave propagation into the corrugated structures. The most popular technique is the coupled mode theory, which relates the counter propagating fields in the grating by two differential equations. Matrix methods are also used for the purpose of grating analysis such as the effective index and the transfer matrix methods [3,4,5].

It is assumed that the effective refractive index of a uniform Bragg grating for the modes of interest to be described as in the following equation:

$$\delta n_{eff}(z) = \overline{\delta n_{eff}}(z) \left(1 + s \cos\left(\frac{2\pi}{\Lambda} z + \varphi(z)\right)\right) \quad (1)$$

where s is the fringe visibility associated with the index change, Λ is the grating period, $\varphi(z)$ is relation to the grating chirp, and $\overline{\delta n_{eff}}$ is the “dc” index change spatially averaged over the grating period, or the slowly varying envelop of the grating. For step index fiber and uniform

induced index change across the core δn_{co} , then $\delta n_{eff} \approx \Phi \delta n_{co}$ where Φ is the core power confinement factor for the mode of interest and can be found from:

$$\Phi = \frac{b^2}{V^2} \left[1 - \frac{J_l^2(V\sqrt{1-b})}{J_{l+1}(V\sqrt{1-b})J_{l-1}(V\sqrt{1-b})} \right] \quad (2)$$

where l is the azimuthal order of the mode and J is the Bessel function of the first kind.

For uniform grating, the interaction happens between the range of wavelength near to the Bragg wavelength, between a mode of amplitude $A(z)$ and a counter propagating mode with amplitude $B(z)$. From the coupled mode theory, the differential equations can be simplified to the following:

$$\frac{dA^+}{dz} = i\xi^+ A^+(z) + i\kappa B^+(z) \quad (3)$$

$$\frac{dB^+}{dz} = -i\xi^+ B^+(z) - i\kappa^* A^+(z) \quad (4)$$

where $A^+(z)$ and $B^+(z)$ are defined as:

$$A^+(z) = A(z) \exp(i\delta_d z - \frac{\varphi}{2}) \quad (5)$$

$$B^+(z) = B(z) \exp(-i\delta_d z + \frac{\varphi}{2}) \quad (6)$$

and ξ^+ is the ‘‘dc’’ self coupling coefficient defined as:

$$\xi^+ = \delta_d + \xi - \frac{1}{2} \frac{d\varphi}{dz} \quad (7)$$

with δ_d being the detuning, which is independent of z , and is defined in the following equation:

$$\delta_d = 2\pi n_{eff} \left[\frac{1}{\lambda} - \frac{1}{\lambda_d} \right] \quad (8)$$

here $\lambda_d = 2n_{eff}\Lambda$ is the peak reflection wavelength for infinitesimally weak index of refraction change grating ($\delta n_{eff} \rightarrow 0$). A complex coefficient ξ can describe the absorption loss in the grating where the power loss coefficient will be given by $a = 2\text{Im}(\xi)$. For a single mode Bragg reflection the following simplified relations are found:

$$\xi = \frac{2\pi}{\lambda} \overline{\delta n_{eff}} \quad (9)$$

$$\kappa = \kappa^* = \frac{\pi}{\lambda} \overline{s \delta n_{eff}} \quad (10)$$

If the grating is uniform along the z direction, then δn_{eff} is constant and $\frac{d\varphi}{dz} = 0$ because there is no chirp. Thus κ, ξ , and ξ^+ are constants. This simplifies the differential equations into coupled first order ordinary differential equations with constant coefficients. With boundary conditions given, they can be solved to find $A^+(z)$ and $B^+(z)$. For a grating with length L the reflectivity can be found assuming a forward propagating wave incident from $z = -\infty$, while requiring that no backward propagating wave exist for $z \geq L/2$. The amplitude $\rho = B^+(-L/2)/A^+(-L/2)$ and the power reflection coefficients $R = |\rho|^2$ can be shown to be:

$$\rho = \frac{-\kappa \sinh \sqrt{(\kappa L)^2 - (\xi^+ L)^2}}{\xi^+ \sinh \sqrt{(\kappa L)^2 - (\xi^+ L)^2} + i\sqrt{\kappa^2 - \xi^{+2}} \cosh \sqrt{(\kappa L)^2 - (\xi^+ L)^2}} \quad (11)$$

and

$$R = \frac{\sinh^2 \sqrt{(\kappa L)^2 - (\xi^+ L)^2}}{-\frac{\xi^{+2}}{\kappa^2} + \cosh^2 \sqrt{(\kappa L)^2 - (\xi^+ L)^2}} \quad (12)$$

There is no simple analytical solution for non-uniform grating because the variables cannot be separated since they all affect the transfer function. In the transfer matrix method, the coupled mode equations are used to calculate the output field for a short section of the grating, assuming that short section is a uniform grating. Identifying 2 by 2 matrix for each section, and then multiplying them together will provide one 2 by 2 matrix that describes the whole grating. The grating can be divided into M sections and then M matrices with A_k^+ and B_k^+ being the field amplitudes after traversing the section K . Starting from the boundary conditions: $A_0^+ = A^+(L/2) = 1, B_0^+ = B^+(L/2) = 0$, the final matrix component $A_M^+ = A^+(-L/2)$ and $B_M^+ = B^+(-L/2)$ can be found as:

$$\begin{pmatrix} A_k^+ \\ B_k^+ \end{pmatrix} = T_k \begin{pmatrix} A_{k-1}^+ \\ B_{k-1}^+ \end{pmatrix} \quad (13)$$

where the matrix T_k for Bragg gratings is given by:

$$T_k = \begin{pmatrix} \cosh(\Omega dz) - i \frac{\zeta^+}{\Omega} \sinh(\Omega dz) & -i \frac{\kappa}{\Omega} \sinh(\Omega dz) \\ i \frac{\kappa}{\Omega} \sinh(\Omega dz) & \cosh(\Omega dz) + i \frac{\zeta^+}{\Omega} \sinh(\Omega dz) \end{pmatrix} \quad (14)$$

dz is the length for the k th uniform section, ζ^+ and κ are the local coupling coefficients for the k th section, and

$$\Omega = \sqrt{\kappa^2 - \zeta^{+2}} \quad (15)$$

The total grating structure can be expressed as:

$$\begin{pmatrix} A_M^+ \\ B_M^+ \end{pmatrix} = T_M T_{M-1} T_{M-2} \dots T_k \dots T_1 \begin{pmatrix} A_0^+ \\ B_0^+ \end{pmatrix} \quad (16)$$

III. SIMULATION AND RESULTS

MathCad is used to develop step-wise linear functions that approximate the Raised cosine with different values of error. These functions are used as the apodization profile with IFO-Gratings Software which is used to calculate the performance parameters of the resulted gratings at each value of error.

Segmenting the apodization function will result in an error in reference to an ideal apodization profile. The number of segments depends on the length of the grating, the apodization profile, and the error allowed. While the effect of the grating length and the apodization profile is evident, the effect of error is more complex. The error is defined as the absolute value of the maximum difference between the segmented and ideal apodization profiles within each segment as shown in Fig. 1.

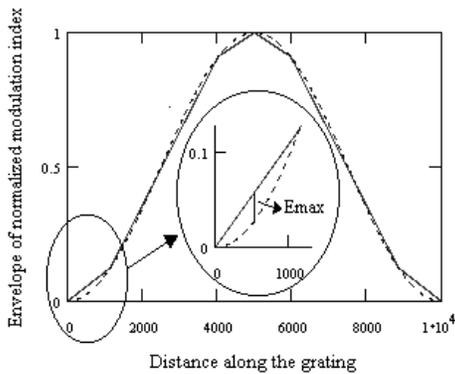


Figure 1: The error E_{max} of the segmented apodization profile

The execution of the following mathematical relation explains the method used to calculate the approximated profile. Given the length of the grating L , the normalized Gaussian function is given by:

$$t(x) = e^{-\alpha(x-L/2)^2} \quad (17)$$

where L is the length of the grating, and x is the distance along the grating.

The program starts by reading the first given value for the maximum error ($E_{max}=0.001$) and with the first point, $(a_1, t(a_1))$, on the function $t(x)$, which is $(0, t(0))$. Let us refer to the equation describing the i th segment by $s_i(x)$ where $s_i(x)$ is the equation of the straight line stretches between $(a_i, t(a_i))$ and $(b_i, t(b_i))$. The program as described below computes the second point. From basic calculus, the equation can be written as:

$$s_i(x, b_i) = P_i(x - a_i) + t(a_i), \quad (18)$$

where P_i is given by:

$$P_i(b_i) = \frac{t(b_i) - t(a_i)}{b_i - a_i}. \quad (19)$$

Next, the point $x_{i,o}$ at which the difference between the smooth function and the i th segment function ($t(x) - s_i(x)$) is a maximum value is determined, i.e.:

$$x_{i,o}(b_i) = \text{root} \frac{d}{dx} (t(x) - s_i(x, b_i)) \quad (20)$$

Finally, b_i is determined by solving the following equation:

$$b_i = \text{root} (|t(x_{i,o}(b_i)) - s_i(x_{i,o}(b_i))| - E_{max}) \quad (21)$$

The next segment is calculated by substituting:

$$(a_{i+1}, t(a_{i+1})) = (b_i, t(b_i)). \quad (22)$$

The procedure is repeated until the calculated $b_i > L/2$. Then this value is replaced by $L/2$ to keep the approximated apodization profile symmetrical. This reduces the maximum error in the last segment to be less than the given value and the total number of segments are only even numbers. The symmetrical property makes it possible to calculate the segments in the second half of the grating.

The maximum error value is increased each time to calculate another approximated profile by a step value of 0.001 until the resulted approximated profile has two segments. It is found that the two segment approximated profile

results in an error of 0.87 for the Raised cosine apodization profile.

The fiber and grating parameters used in the simulation are shown in Table 1. These values are fixed for all the approximated profiles under test.

TABLE 1: COMMON STUDY PARAMETERS

Parameter	Value
Fiber type	SMF-28
Central wavelength (nm)	1550
Grating length L (mm)	10
Chirp	No chirp
Slanted	No slanted
Grating shape	Sinusoidal

Fig. 2 shows the integration of the error function, and the maximum reflectivity as a function of the maximum error. The change in the reflectivity is related to the area between the smooth and the approximated profiles, which is the integration of the error function along the grating length. For negative values of the integration function, the maximum reflectivity will increase and vice versa.

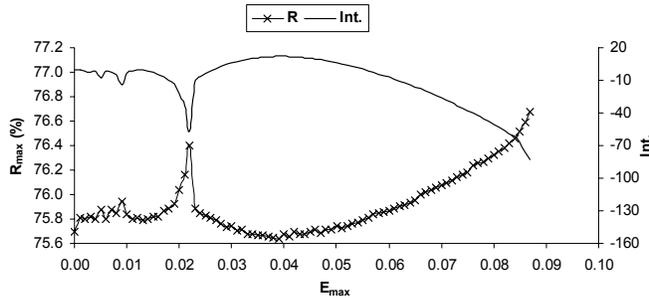


Figure 2: The maximum reflectivity and the integration of the error as a function of maximum error for approximated Gaussian apodized FBGs

Fig. 3 shows the SLSR and the integration function as a function of error. The SLSR is directly proportional to the integration

function in the error range of 0-0.014. Sharp transition reduces the SLSR around 10 dB, and the curve after this point behaves opposite to the integration function. The shape of the spectrum shows the splitting of the main lobe, which explains the behavior of sharp SLSR reduction.

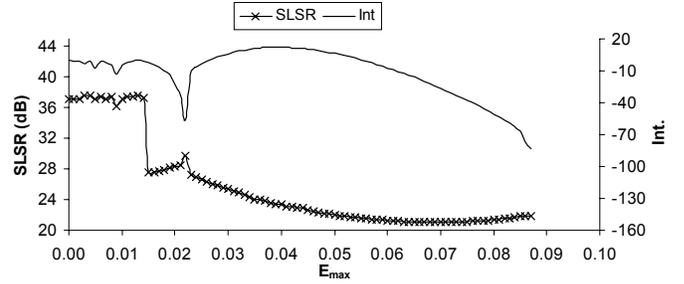


Figure 3: The SLSR and integration of error for the approximated Gaussian apodized FBGs as a function of error.

The reflection spectrum for two apodized gratings is shown in Figure 4, one with maximum error of 0.014, and the second with 0.08. For maximum error larger than 0.014, the main lobe is divided into three lobes because of the increase in the index modulation. This splitting decreases the SLSR by a value of around 10dB because now the first lobe is the one coming out from the main lobe.

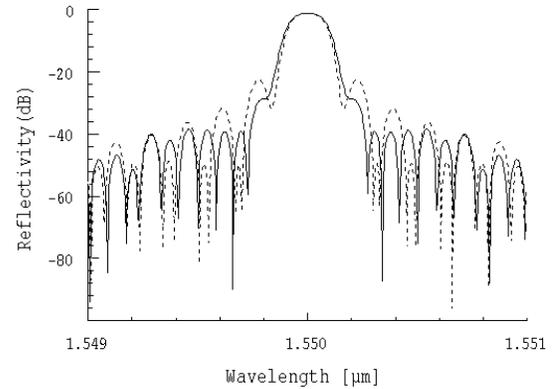


Figure 4: Reflectivity spectra for segmented Gaussian apodized normal FBGs with maximum error of 0.014 (solid line) and 0.08 (dashed line).

Fig. 5 shows the FWHM bandwidth as a function of error. The curve is slowly decreasing because of the main lobe splitting with bandwidth difference between the smooth profile and two-segment profile of 0.01 nm. The decrement is nearly linear along the whole range of error. At the local maxima of the error function integration, the bandwidth increases above the linear path as can be seen at maximum error of 0.022. This

behavior is related to the index modulation increase represented by the integration function.

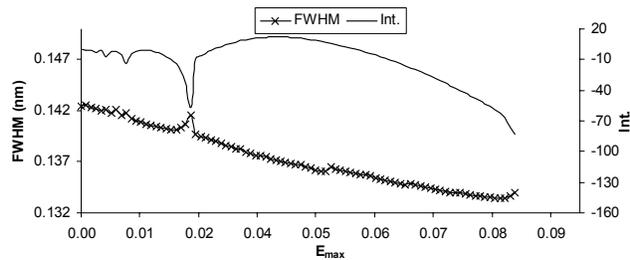


Figure 5: The approximated Gaussian apodized FBGs FWHM bandwidth and the integration of error as a function of maximum error.

Conclusion: SLSR is maintained for normal FBG apodized with linearly approximated Gaussian apodization profile that relaxes the implementation. The approximated Gaussian apodized FBGs has close performance parameters to that of the smooth profile if the profile is approximated with the maximum error of less than 0.014. The SLSR decreases sharply to 10dB less for any gratings apodized with the approximated Gaussian profile with maximum error more than 0.014. The number of segments for the approximated profile at this maximum error is 10 segments, which is considered small number compared to the smooth function and easier for implementation. The maximum reflectivity is enhanced by about 1dB with the approximated profiles in this range of maximum error.

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